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Hierarchical Order of Galilei and Lorentz Invariance in the Structure of Matter

F. Winterberg[1]

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The structure of matter shows a hierarchical order: (1) from Lorentz invariance in high-energy physics; (2) to Galilei invariance in the low-energy nonrelativistic limit of high-energy physics; and (3) again to Lorentz invariance in condensed matter physics (where the velocity of sound takes the place of the velocity of light). The hierarchical order can be continued downward further to: (4) non-relativistic (velocity small compared to the velocity of sound) condensed matter excitons, obeying Galilei invariance; and (5) to excitonic matter obeying Lorentz invariance with an excitonic matter sound velocity. It was previously conjectured that Lorentz invariance of high-energy physics is preceded by Galilei invariance at the Planck scale. Still further, the conjectured Galilei invariance at the Planck scale may be the result of an underlying five-dimensional non-Euclidean conform invariant metric structure, with three spatial and two time dimensions, compactified onto three spatial and one time dimension.

1. INTRODUCTION

Voight (1887) showed as far back as 1887 that any wave equation of the form

$$-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\nabla^2\phi=0 \tag{1.1}$$

obeys Lorentz invariance. This means that Lorentz invariance is also valid for acoustic waves, provided the velocity of light is replaced with the velocity of sound. The significance of Lorentz invariance in acoustics was recognized by Prandtl (in a rule named after him), often used for gas dynamic calculations. Küssner (1940), for example, applies the Lorentz

[1]Desert Research Institute, University and Community College System of Nevada, Reno, Nevada 89506.

transformation to compute the wave field of a moving sound source. The argument may be made that Lorentz invariance of the sound field is not "real," because it depends on a gas which is its carrier, consisting of molecules which obey nonrelativistic Newtonian mechanics. But if this position is taken, one may ask if Lorentz invariance observed in high-energy physics is unreal in a likewise sense. Theoretical arguments suggest a unification of all forces (except gravity) at the very high energy of $\sim 10^{16}$ GeV, corresponding to a length of $\sim 10^{-30}$ cm. A unification with gravity is expected at an energy of $\sim 10^{19}$ GeV (at a length of $\sim 10^{-33}$ cm) not very far away (on a logarithmic scale) from the energy of $\sim 10^{16}$ GeV. A case can therefore be made that the observed Lorentz invariance in high-energy physics (with energies far below 10^{16}–10^{19} GeV) is in reality the result of a hidden substratum or aether made up from very small discrete objects, obeying at very high energies a nonrelativistic law of motion. According to this conjecture, elementary particles would have to be understood as quantized quasi-particles, like the quasiparticles known from condensed matter physics. In the especially simple case of a gas or liquid, the quasiparticles are phonons. For these quasiparticles, the term "very high energy" would mean an energy corresponding to a wavelength near the de Broglie wavelength of the molecules making up the gas or liquid. Under normal conditions, the energy of the phonons is much smaller, very much like the observed elementary particles in high-energy physics, which have an energy well below $\sim 10^{16}$ GeV. An ideal gas can therefore serve as a model to demonstrate how Lorentz invariance can be derived from an underlying Galilei-invariant structure.

Comparing condensed matter physics with elementary physics, phonons correspond to photons, and excitons to fermions, but the similarity does not end here. Even the Higgs mechanism has a condensed-matter-physics counterpart in the Landau–Ginzburg equations of superfluidity, and the fractional quantized Hall effect can be explained in terms of fractional charges, mimicking fractionally charged quarks. In spite of many striking analogies between condensed matter and high-energy physics, there is one important difference. Whereas the condensed matter state has a nonzero mass density, the mass density of the vacuum is zero (respectively, appears to be indistinguishable from zero).

Prior to Einstein, it was shown by Lorentz and Poincaré that all relativistic effects can be explained by postulating the existence of an aether, because if electromagnetic waves in the aether rest frame obey Maxwell's equations, and if all material objects are held together by electromagnetic forces, or forces acting like them, bodies in absolute motion against the aether with a velocity v would suffer a real contradiction by the factor $(1 - v^2/c^2)^{1/2}$ and clocks made from such bodies would go slower by the same factor (Prokhovnik, 1967). In this interpretation, Lorentz invariance is seen as an illusion caused by true physical deformations, but because the vacuum has no mass, this older pre-Einstein Lorentz–Poincaré theory of relativity lost ground, giving way to Einstein's theory of relativity explaining Lorentz invariance as a purely kinematic symmetry of space and time. The advent of quantum mechanics, however, showed that the problem of a massless vacuum is a much more subtle one than Einstein could have possibly thought. Assuming the correctness of the theory of relativity as a space-time symmetry, quantum theory leads to an infinite vacuum energy caused by the zero-point vacuum fluctuations of the electromagnetic (and other) fields. An infinite vacuum energy should result in infinite gravitational fields, which are obviously not observed.

In condensed matter physics, we have the phenomenon of electric charge neutrality. That it is highly perfect can be illustrated by the force of $\sim 10^{16}$ tons needed to separate the charges in 1 cm^3 of condensed matter. In analogy to this electric charge neutrality, one may entertain the hypothesis that the vacuum has a second negative mass component canceling the huge positive mass component of the zero-point energy, leading to the observed "mass neutrality" of the vacuum. According to Planck (1899), all measurable quantities of physics should be expressed by the three fundamental constants, h (Planck's constant), c (velocity of light), and G (Newton's constant), leading to the Planck length $\sim 10^{-33}$ cm and Planck mass $\sim 10^{-5}$ g as expressed in terms of these constants. General relativity in combination with quantum mechanics suggests that the vacuum should be densely filled with Planck masses, each having the spatial extension of a Planck length. Because the resulting vacuum mass density would be huge ($\sim 10^{95}$ g/cm^3), it was suggested by Sakharov (1968) that it is compensated by "ghost particles." Since in a nonrelativistic theory the Hamilton operator commutes with the particle number operator, I have put forward the hypothesis that Sakharov's "ghost particles" are negative Planck masses, and that the vacuum is densely filled with an equal number of positive and negative Planck masses, with both the positive and negative Planck masses obeying an exactly nonrelativistic law of motion. In such a model, the total number of each species of Planck masses is conserved and no decay of positive into negative masses is possible. A cube with a side length of 1 F and densely filled with positive Planck masses alone would have a mass about equal to the mass of the entire known universe, demonstrating the even more perfect mass neutrality of the vacuum if compared with the electric charge neutrality of condensed matter. The interactions between the Planck masses are assumed local, repulsive between Planck masses of equal and attractive between those of opposite sign, with both components forming a superfluid, taking the place of the pre-Einstein

Table I

Some Correspondences between High Energy and Condensed Matter Physics

High-energy physics	Condensed matter physics
Lorentz invariance with velocity of light	Lorentz invariance with velocity of sound
Planck energy cutoff	Debye energy cutoff
Mass neutrality of vacuum	Electric charge neutrality of condensed matter
Photons, gravitons, etc.	Phonons, rotons
Fermions	Excitons
Dirac hole theory	Electron holes in solids
Goldstone boson	Plasmon
Higgs field	Landau–Ginzburg field of superfluidity
Fractional charges of quarks	Fractional electron charges in fractional quantum Hall effect
Parity anomaly	Quantum Hall effect without Landau levels

aether of 19th century physics. The quasiparticles of this quantum aether would be what one calls elementary particles. Lorentz invariance in elementary particle physics would be a dynamic symmetry to follow from collective modes of the Planck aether, obeying the classical wave equation (1.1.). In a similar way as the existence of excitons (and other quasiparticles) in condensed matter physics depends on the coexistence of equal and opposite electric charges, the elementary particles in high-energy physics would have to be understood to result from the coexistence of positive and negative Planck masses. Only a configuration of interacting positive and negative Planck masses has the potential to result by partial mutual compensation in the much smaller mass of elementary particles if compared with the Planck mass.

In Table I, a number of correspondences between high-energy and condensed matter physics have been put together side by side.

2. FROM GALILEI TO LORENTZ INVARIANCE

It was shown by Selleri (1990) that in addition to the Galilei and Lorentz transformations there is an infinite number of transformations interpolating between both, but that only the Galilei and Lorentz transformations form a group. Selleri's considerations were purely kinematic, but it is easy to show how the transition from one to the other group can be understood dynamically.

To demonstrate this transition, we consider N interacting particles of mass m_0 described by a nonrelativistic many-body Galilei-invariant Hamiltonian function

$$H(p_1, p_2, \ldots, q_1, q_2, \ldots) = \sum_i^N \frac{p_i^2}{2m_0} + V(q_1, q_2, \ldots) \tag{2.1}$$

where V is a nonrelativistic interaction potential. The transition to quantum mechanics is, as usual, done by putting $p_i = (\hbar/i)\,\partial/\partial q_i$. In the limit $N \to \infty$, and if the interaction potential V between the particles can be described by a delta function, the many-body Schrödinger equation for the Hamilton operator constructed from the Hamiltonian function (2.1) can be replaced by the operator field equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_0} \nabla^2 \psi + f^2 \psi^\dagger \psi \psi \tag{2.2}$$

where f is a coupling constant and where the operators $\psi, \psi^\dagger$ obey the commutation relations

$$\begin{aligned} [\psi(\mathbf{r})\, \psi^\dagger(\mathbf{r}')] &= \delta(\mathbf{r} - \mathbf{r}') \\ [\psi(\mathbf{r})\, \psi(\mathbf{r}')] &= [\psi^\dagger(\mathbf{r})\, \psi^\dagger(\mathbf{r}')] = 0 \end{aligned} \tag{2.3}$$

In solving the nonlinear operator equation (2.2), we make the Hartree approximation (Nozieres, 1966)

$$\begin{aligned} \varphi &= \langle \psi \rangle \\ \varphi^* &= \langle \psi^\dagger \rangle \\ \varphi^* \varphi^2 &\simeq \langle \psi^\dagger \psi \psi \rangle \end{aligned} \tag{2.4}$$

by which (2.2) becomes a nonlinear Schrödinger equation:

$$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m_0} \nabla^2 \varphi + f^2 \varphi^* \varphi^2 \tag{2.5}$$

With Madelung's (1926) transformation

$$\begin{aligned} n &= \varphi^* \varphi \\ n\mathbf{v} &= -\frac{i\hbar}{2m_0} [\varphi^* \nabla \varphi - \varphi \nabla \varphi^*] \end{aligned} \tag{2.6}$$

we obtain the hydrodynamic form of (2.5):

$$\begin{aligned}\frac{d\mathbf{v}}{dt} &= -\frac{1}{m_0}\operatorname{grad}(V+Q)\\ \frac{\partial n}{\partial t} &+ \operatorname{div}(n\mathbf{v}) = 0\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}\frac{d\mathbf{v}}{dt} &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\\ V &= f^2 n\\ Q &= -\frac{\hbar^2}{2m_0}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\end{aligned} \tag{2.8}$$

with V the "regular" and Q the "quantum" potential.

Introducing the velocity potential

$$\mathbf{v} = -\operatorname{grad}\phi \tag{2.9}$$

and the function

$$W(n) = \frac{1}{nm_0}\int (V+Q)\,dn \tag{2.10}$$

we can derive the hydrodynamic equations (2.7) from the Lagrange density

$$\mathscr{L}_1 = nm_0[\dot{\phi} - \tfrac{1}{2}(\operatorname{grad}\phi)^2 - W(n)] \tag{2.11}$$

Variation with regard to n leads to

$$\dot{\phi} - \frac{1}{2}(\operatorname{grad}\phi)^2 - \frac{1}{m_0}(V+Q) = 0 \tag{2.12}$$

which is Bernoulli's equation. Euler's equation (2.7) is obtained from (2.12) by taking the gradient. Variation of (2.11) with regard to ϕ leads to

$$-\dot{n} + \operatorname{div}(n\operatorname{grad}\phi) = 0 \tag{2.13}$$

which is the continuity equation.

The Lagrange density $\mathscr{L}_1$ is completely nonrelativistic and therefore Galilei invariant. The transition to a relativistic invariant Lagrange density is accomplished by two approximations: (1) keeping in $\mathscr{L}_1$ only terms quadratic in n, ϕ or the product of n and ϕ; (2) omitting the quantum potential.

The first approximation implies small-amplitude disturbances, and the second approximation long wavelengths. In making these two approximations, one obtains the Lagrange density

$$\mathscr{L}_2 = n\dot{\phi} - \frac{n_0}{2}(\nabla\phi)^2 - \frac{1}{2}\frac{f^2 n^2}{m_0} \tag{2.14}$$

where n_0 is a constant density from which small departures are permitted. Variation of (2.14) with regard to n and ϕ leads to

$$\begin{aligned} \dot{\phi} - (f^2/m_0)n &= 0 \\ -\dot{n} + n_0 \nabla^2\phi &= 0 \end{aligned} \tag{2.15}$$

Eliminating n from these two equations leads to the wave equation

$$-\frac{1}{c^2}\ddot{\phi} + \nabla^2\phi = 0 \tag{2.16}$$

where

$$c^2 = n_0 f^2/m_0 \tag{2.17}$$

The wave equation (2.16) can be derived from the Lagrange density

$$\mathscr{L}_3 = \tfrac{1}{2}[\dot{\phi}^2 - c^2(\nabla\phi)^2] \tag{2.18}$$

which, unlike $\mathscr{L}_1$, is Lorentz invariant. If applied to acoustics, Lorentz invariance would break down for wavelengths smaller than the mean free path of the gas molecules. The way a system of particles can undergo a transition from Galilei to Lorentz invariance is, therefore, established on purely dynamical grounds.

3. FROM GENERAL RELATIVITY TO GALILEI INVARIANCE

We now show how a transition from general relativity to a Galilei-invariant field theory is possible, at least in a rudimentary way. General relativity implies the replacement of the Minkowskian space-time metric by a general Riemannian metric expressed through the line element

$$ds^2 = g_{ik}\, dx^i\, dx^k \tag{3.1}$$

with the metric tensor g_{ik} given by a solution of Einstein's gravitational field equations:

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \varkappa T_{ik} \tag{3.2}$$

Instead of Einstein's tensor field equation (3.2), we take its scalar contracted form

$$R = -\varkappa T \tag{3.3}$$

and restrict its solutions to a scalar. We describe this scalar by a conform invariant metric

$$ds^2 = u^2\, ds_0^2 \tag{3.4}$$

where u is a real, but otherwise arbitrary function of space and time, with ds_0 the Minkowskian line element. Using the line element (3.4), one can express the curvature scalar as follows (Gürsey, 1953):

$$R = 6u^{-3}\, \Box u \tag{3.5}$$

If $u = 1$, $R = 0$ and $ds = ds_0$.

To connect u to the field φ of some hypothetical background particles of number density n_0, we put

$$u = \varphi/\sqrt{n_0} \tag{3.6}$$

by which (3.5) becomes

$$\Box \varphi = \frac{R}{6n_0}\, \varphi^3 \tag{3.7}$$

To make the transition to nonrelativistic energies, we put

$$\psi = e^{im_0c^2t/\hbar}\, \varphi \tag{3.8}$$

by which

$$\varphi^2 = \psi^* \psi \tag{3.9}$$

and

$$\frac{\partial^2 \varphi}{\partial t^2} = \left(\frac{\partial^2 \psi}{\partial t^2} - \frac{2im_0c^2}{\hbar} \frac{\partial \psi}{\partial t} - \frac{m_0^2 c^4}{\hbar^2} \psi \right) e^{-im_0c^2t/\hbar} \tag{3.10}$$

The first term in the parentheses of (3.10) can be neglected. For reasons explained below, we also omit the third term. We therefore make the replacement

$$\Box \rightarrow \frac{2im_0}{\hbar} \frac{\partial}{\partial t} + \nabla^2 \tag{3.11}$$

whereby (3.7) becomes

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_0} \nabla^2 \psi + \frac{R\hbar^2}{12m_0 n_0} \psi^* \psi^2 \tag{3.12}$$

It has the same form as (2.2), describing a gas of particles with contact-type interactions. With the coupling constant given by

$$f^2 = \frac{R\hbar^2}{12m_0 n_0} \tag{3.13}$$

the wave propagation velocity is according to (2.17) given by

$$c^2 = \frac{R\hbar^2}{12m_0^2} \tag{3.14}$$

Equation (3.12) shares with (3.7) the property that for $\psi = \text{const}$, $R = 0$. This condition could not be met if we had not omitted the third term in (3.10). But now we have a problem, because if $R = 0$, then also $c = 0$. We therefore must repair the damage done in having left out the third term in the parentheses of (3.10). This is done by adding to (3.12) a term on the r.h.s., which thereby becomes

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_0} \nabla^2 \psi - \frac{R\hbar^2}{12m_0} \psi + \frac{R\hbar^2}{12m_0 n_0} \psi^* \psi^2 \tag{3.15}$$

Now, if R is given by (3.14) and if $\psi = \text{const}$, one has $\psi^* \psi = n_0$. The second term added to the r.h.s. of (3.15) can be given a simple interpretation which can be seen as follows: The Schrödinger operator $-(\hbar^2/2m_0)\nabla^2$ must be replaced in a space of constant Gaussian curvature K by

$$-\frac{\hbar^2}{2m_0} \nabla^2 \to -\frac{\hbar^2}{2m_0} \nabla^2 + \frac{\hbar^2 K}{2m_0} \tag{3.16}$$

In three-dimensional space, the curvature scalar R is expressed by K through (Eisenhart, 1926)

$$K = -R/6 \tag{3.17}$$

whereby $\hbar^2 K/2m_0 = -R\hbar^2/12m_0$. The second term in (3.15), therefore, can be seen as the effect of a negative space curvature, which for $\psi = \text{const}$ is equal and opposite to the space curvature resulting from the third term. Both curvatures combined result in a flat (three-dimensional) space.

The term added to the r.h.s. of (3.15) can be accounted for in Einstein's contracted field equation (3.3) by making the following change in the expressions for R:

$$R = \frac{6n_0}{\varphi^3}\left[\square\varphi + \left(\frac{m_0 c}{\hbar}\right)^2 \varphi\right] \tag{3.18}$$

This change can be interpreted to result from the compactification of one time dimension of a five-dimensional field equation, with three spatial and two time coordinates, and with the compactification length equal to $\hbar/m_0 c$. We note that for a space of constant curvature, (3.18) has the same form as the Higgs field equation, which in the standard model is responsible for generating mass. Without the mass term added to the r.h.s. of (3.18), the particles associated with this field would have zero rest mass, as it is to be expected for spin-zero gravitons of the contracted Einstein equation (3.3) for $R = -\varkappa T = \text{const}$. From this perspective, the Higgs field can be interpreted to arise from the compactification of a five-dimensional scalar Einstein gravitational field equation.

Applying these results to the Planck aether model, which assumes that space is densely occupied with an equal number of positive and negative Planck masses, the number density of the positive and negative Planck masses is $n_0 = 1/2r_0^3$, where r_0 is the Planck length. If m_0 is set equal the Planck mass, one has, because of $m_0 r_0 c = \hbar$, and by setting the value of c in (3.14) equal to the velocity of light,

$$R = \frac{12}{r_0^2} \tag{3.19}$$

This very large value of the space curvature, consistent with the Planck length as the gravitational radius of a Planck mass, demonstrates the necessity for a compensating space curvature, which in the Planck aether model is provided by the negative Planck masses. The compactification length now becomes equal to the Planck length, as in the five-dimensional Kaluza–Klein theory.

Inserting into (3.18) the value for R given by (3.19) and with $n_0 = 1/2r_0^3$, one obtains from (3.18) the nonlinear relativistic wave equation

$$\square\varphi + \frac{\varphi}{r_0^2} - 4r_0\varphi^3 = 0 \tag{3.20}$$

According to (3.10), we set in the nonrelativistic limit without any omissions

$$\square \to \frac{2im_0}{\hbar}\frac{\partial}{\partial t} + \nabla^2 + \left(\frac{m_0 c}{\hbar}\right)^2 \tag{3.21}$$

and furthermore express φ by ψ. It transforms (3.20) into

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_0} \nabla^2 \psi - m_0 c^2 \psi + n_0 m_0 c^2 \psi^* \psi^2 \tag{3.22}$$

With R given by (3.19), it is the same as (3.15). It can be interpreted as the field equation describing the positive Planck masses, by setting $\psi \to \psi_+$, and correspondingly by setting $-m_0 c^2 = -n_0 m_0 c^2 \psi_-^* \psi_-$, where ψ_- is the field of the negative Planck masses. It is the second term on the r.h.s. of (3.22) which brings to light the need for negative Planck masses. With a similar equation valid for negative Planck masses, an equation for both the positive and negative Planck masses can be written down:

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_0} \nabla^2 \psi_\pm + 2\hbar c r_0^2 (\psi_+^* \psi_+ - \psi_-^* \psi_-) \psi_\pm \tag{3.23}$$

It is this equation which was previously used as a model for the hypothetical Planck aether.

4. DISCUSSION

We have shown that the simple model of a scalar field can serve to illustrate how Lorentz invariance can be understood as a dynamic symmetry to follow from an underlying discrete Galilei-invariant structure. Taking the quantized Lorentz-invariant Klein–Gordon equation as the starting point instead, one obtains a discrete set of bosons. In the limit of nonrelativistic energies they can be assembled into a gas of interacting particles, leading to an acoustic-type wave equation, again exhibiting Lorentz invariance, albeit with a different reference velocity. Quantizing the acoustic equation, in turn, leads to particles which in this case are phonons, but because these particles are of the zero-rest-mass type, there is no nonrelativistic low-energy limit from which a lower hierarchy could be deduced. This is possible in condensed matter, which has a much richer number of quasiparticles, in particular, nonzero-rest-mass fermionic excitons. In the low-energy limit, these excitons can condense into a novel form of matter, called excitonic matter. In turn, it can have its own quasiparticles and a velocity of sound associated with them. The existence of excitonic matter has been conjectured theoretically and its experimental verification by powerful laser discharges has been proposed (Haken, 1983).

We can see at least five hierarchies: (1) Lorentz-invariant high-energy particle physics; (2) Galilei-invariant low-energy atomic physics; (3) Lorentz-invariant condensed matter quasiparticle physics; (4) Galilei-invariant low-energy excitonic particle physics; and (5) Lorentz-invariant condensed excitonic matter physics. In addition, there are other collective structures not included in this list, having their own respective invariant properties, notably condensed nuclear matter physics and collective stellar and galactic structures.

Because the Planck aether model describes physics at the Planck scale at which gravitational effects become predominant, it is interesting to ask if (in the spirit of Einstein) it can be derived from a metric structure at or above this scale. We showed that this is possible. It was Einstein's hope that all elementary particles can somehow be explained by gravity alone, and that all of physics can be reduced to a curved space-time structure.

In light of the conjectured underlying five-dimensional space-time structure above the Planck scale, the old controversy about the existence or nonexistence of the aether attains a new perspective. In the underlying five-dimensional space-time structure there is no aether, however, which is generated by compactification of one time dimension onto the Planck length. It is this compactification which generates nonzero-rest-mass particles, with a mass equal to the Planck mass.

With the curvature scalar R related to the Gaussian curvature K in a space of n dimensions by (Eisenhart, 1926)

$$R = -(n-1)nK \tag{4.1}$$

where for $n=4$ one has $R = -12K$, one can, in five dimensions, even write down the fundamental law in a very compact form:

$$K = -1/r_0^2 \tag{4.2}$$

The same law holds if the noncompactified space is six dimensional, with three spatial and three time dimensions, with the compactification of

Table II

Hierarchical Structure of Matter, Established and Conjectured

Scale	Symmetry
Hyper-Planck	Conformal five-dimensional Einstein
Planck	Galilei
High-energy elementary particle physics	Lorentz
Low-energy atomic physics	Galilei
Condensed matter physics	Lorentz
Condensed matter low-energy excitons	Galilei
Excitonic matter	Lorentz

two time dimensions generating the Planck mass. Assuming an equal number of spacelike and timelike dimensions has the appeal for a more symmetric fundamental law.

In Table II we have put together the hierarchical structure of the known and conjectured fundamental structures of matter.

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